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ABSTRACT

The conclusions set forth in the present study provide a generalization of the Chapman-Enskog Method; this generalization makes it possible to study gas mixtures with internal degrees of freedom and chemical reactions.

The study differs from works which have been published recently (Ref 1, Ref 2, and Ref 3) in two ways:

(1) A study is made of gas mixtures in which chemical exchange reactions can occur (i.e., when two particles collide, two and only two particles are produced);

(2) A different system of macroscopic parameters is selected, by which the distribution functions are represented; in this connection, new macroscopic equations for determining these parameters are developed.

The point of departure for our study is Ref. 4 and Ref. 5. From the former, we derive an expression for the collision integral; from the latter, we obtain the form for the equilibrium solution of the corresponding system of Boltzmann equations. The notations from Ref. 4, Ref. 5 and Ref. 6 will be used in the article.

1. ZERO APPROXIMATION. SEPARATION OF $D_i f_i$

Let us write the system of Boltzmann equations in the form *:

$$\frac{\partial f_i}{\partial t} + \mathbf{u}\cdot\frac{\partial f_i}{\partial \mathbf{r}} = \frac{1}{2}\sum_{k,l,n}\iiint\limits_{-\infty}^{+\infty}(f_k f_e' - f_i f_n) K_{in}^{kl} d\mathbf{u}_1' d\mathbf{u}_2' d\mathbf{u}_1. \tag{1.1}$$

According to Enskog, we can set $f_i = \sum_{m=0}^{\infty} f_i^{(m)}$, where each approximation of $f_i^{(m)}$ is determined from the linear integral equation:

$$\xi_i^{(0)} = J_i^{(0)} = 0, \tag{1.2}$$

$$\xi_i^{(m)} = J_i^{(m)} + D_i^{(m)} = 0 \quad (m > 0). \tag{1.3}$$

Here

$$J_i^{(m)} = \frac{1}{2}\sum_{k,l,n}\iiint\limits_{-\infty}^{+\infty}\left(f_l^{(0)} f_n^{(m)} + \ldots + f_l^{(m)} f_n^{(0)} - f_k^{(0)} f_l^{(m)} - \ldots - f_k^{(m)} f_l^{(0)}\right) K_{in}^{kl} d\mathbf{u}_1' d\mathbf{u}_2' d\mathbf{u}_1,$$

and

$$D_i^{(m)} = D_i^{(m)}\left(f_i^{(0)}, \ldots, f_i^{(m-1)}\right).$$

The zero approximation of the distribution function is obtained from equation (1.2) in the form:

$$f_i^{(0)}(\mathbf{r}, \mathbf{u}, t) = \alpha \exp\left\{\sum_\lambda K_{\lambda i}\mu_\lambda - \frac{1}{\vartheta}\left(\frac{1}{2} m_i U^2 + \varepsilon_i\right) + \mathbf{v}\cdot\mathbf{J}_i\right\}, \tag{1.4}$$

where J_i is the average internal moment of the i^{th} particle for the given state. The coefficients α, μ_1, ... , μ_r, ϑ, and $\vec{\nu}$ are arbitrary functions of r and t. Just as in (Ref. 5), we selected them in such a way that they could be determined from (r + 8) balance equations:

$$\sum_i \int\limits_{-\infty}^{+\infty} f_i^{(0)}(\mathbf{r}, \mathbf{u}, t)\, d\mathbf{u} = N_0, \tag{1.5}$$

$$\sum_i \nu_{\lambda i}\int\limits_{-\infty}^{+\infty} f_i^{(0)}(\mathbf{r}, \mathbf{u}, t)\, d\mathbf{u} = N_\lambda \quad (\lambda = 1, \ldots, r), \tag{1.6}$$

$$\sum_i \int\limits_{-\infty}^{+\infty}\left(\frac{m_i \mathbf{u}^2}{2} + \varepsilon_i\right) f_i^{(0)}(\mathbf{r}, \mathbf{u}, t)\, d\mathbf{u} = E, \tag{1.7}$$

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* Just as in (Ref. 5), the principle of detailed balance is assumed to hold here.

$$\sum_i J_i \int_{-\infty}^{+\infty} f_i^{(0)}(\mathbf{r}, \mathbf{u}, t)\, d\mathbf{u} = \mathbf{I}_0. \tag{1.8}$$

$$\sum_i \int_{-\infty}^{+\infty} m_i \mathbf{u} f_i^{(0)}(\mathbf{r}, \mathbf{u}, t)\, d\mathbf{u} = \rho \mathbf{u}_0. \tag{1.9}$$

Here $I_0(r, t)$ is the total internal moment. We determine the concrete form of $D_i^{(m)}$ in order to obtain the following approximations. To do this, we make use of the transport equation:

$$\frac{DN_0}{Dt} + N_0 \frac{\partial}{\partial \mathbf{r}} u_0 + \frac{\partial}{\partial \mathbf{r}} \cdot \sum_i (n_i \widetilde{\mathbf{U}}_i) = 0, \tag{1.10}$$

$$\frac{DN_\lambda}{Dt} + N_\lambda \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{u}_0 + \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{R}_\lambda = 0, \quad \lambda = 1, \ldots, r, \tag{1.11}$$

$$\frac{DE_0}{Dt} + E_0 \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{u}_0 + \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{q} + \overline{\mathbf{P}} : \frac{\partial}{\partial \mathbf{r}} \mathbf{u}_0 = 0, \tag{1.12}$$

$$\frac{D\mathbf{I}_0}{Dt} + \mathbf{I}_0 \left(\frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{u}_0\right) + \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{M} = 0, \tag{1.13}$$

$$\rho \frac{D\mathbf{u}_0}{Dt} + \frac{\partial}{\partial \mathbf{r}} \cdot \overline{\mathbf{P}} = 0. \tag{1.14}$$

Here $\widetilde{U}_i = \frac{1}{n_i} \int_{-\infty}^{+\infty} U f_i dU$ is the diffusion current vector of the i^{th} component; $R_\lambda = \sum_i n_i K_{\lambda i} \widetilde{U}_i$ is the current vector of λ-type atoms; $E_0 = E - \frac{1}{2} \rho u_0^2$ is the total "self" energy of the system; $q = \sum_i q_i = \sum_i \int_{-\infty}^{+\infty} f_i \left(\frac{1}{2} m_i U^2 + \epsilon_i\right) U dU$ is the heat flux vector; $\overline{P} = \sum_i \overline{P}_i = \sum_i \int_{-\infty}^{+\infty} m_i \overline{UU} f_i dU$ is the stress tensor; $\overline{M} = \sum_i \overline{M}_i = \sum_i \overline{n_i J_i \widetilde{U}_i} = \sum_i J_i \int_{-\infty}^{+\infty} U f_i dU$ is the transport tensor of the internal moment.

Equations (1.11) for N_λ (number of λ-type atoms) usually replace the diffusion equations being examined. Their occurrence is connected with the invariants $K_{\lambda i}$. It can be readily seen that the equations for N_λ are simpler than the diffusion equations which are normally used, because the right hand sections are equal to zero. In addition, the number of these equations is less than the number of diffusion equations ($s \geqslant r$), generally speaking.

When the N_λ are found, then the n_i are readily determined with $f_i^{(0)}$. If $N_0, N_1, \ldots, N_r, E, I_{ox}, I_{oy}, I_{oz}$ are considered as independent varables $x_0, x_1, \ldots, x_{r+4}$, and $\alpha, \mu_1, \ldots, \mu_r$, T, ν_x, ν_y, ν_z are considered as functions $y_0, y_1, \ldots, y_{r+4}$ of these variables, then the system of equations (1.5) - (1.8) can be written in the form:

$$F_j(x_0, \ldots, x_{r+4}, y_0, \ldots, y_{r+4}) = 0, \quad j = 0, \ldots, r+4, \tag{1.15}$$

and the system of transport equations (1.10) - (1.14) can be written in the form:

$$\frac{Dx_k}{Dt} + x_k \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{u}_0 + \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{Q}_k + \overline{\mathrm{P}} \delta_{k,r+1} : \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{u}_0 = 0, \quad k = 0, \ldots, r+4, \tag{1.16}$$

$$\rho \frac{D\mathbf{u}_0}{Dt} + \frac{\partial}{\partial \mathbf{r}} \cdot \overline{\mathrm{P}} = 0. \tag{1.17}$$

In equations (1.16) - (1.17) we divide the time derivatives in parts

$$\frac{\partial}{\partial t} = \frac{\partial_0}{\partial t} + \frac{\partial_1}{\partial t} + \ldots,$$

where all $\frac{\partial_m}{\partial t}$ are not derivatives, but operators, whose effect is given by the following relationships:

$$\frac{D_0 x_k}{Dt} = \left(-x_k + \overline{\mathrm{P}} \delta^{(0)}_{k,r+1}\right) \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{u}_0 \quad \left(\frac{D_0}{Dt} = \frac{\partial_0}{\partial t} + \mathbf{u}_0 \frac{\partial}{\partial \mathbf{r}}\right). \tag{1.18}$$

$$\frac{\partial_m x_k}{\partial t} = -\frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{Q}_k^{(m)} - \overline{\mathrm{P}}^{(m)} \delta_{k,r+1} : \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{u}_0 = 0 \quad (m > 0), \tag{1.19}$$

$$\rho \frac{D_0 \mathbf{u}_0}{Dt} = -\frac{\partial}{\partial \mathbf{r}} \cdot \overline{\mathrm{P}}^{(0)} = -\frac{\partial p}{\partial \mathbf{r}}. \tag{1.20}$$

$$\rho \frac{\partial_m \mathbf{u}_0}{\partial t} = -\frac{\partial}{\partial \mathbf{r}} \cdot \overline{\mathrm{P}}^{(m)} \quad (m > 0). \tag{1.21}$$

Here all $Q_k^{(m)}$ and $\overline{P}^{(m)}$ are determined just as in (Ref. 6), only using $f_i^{(m)}$.

According to Enskog, in each approximation we obtain the equations:

$$\frac{1}{2} \sum_{k,l,n} \iiint_{-\infty}^{+\infty} \left[f_l^{(0)} f_n^{(m)} + f_l^{(m)} f_n^{(0)} - f_k^{(0)} f_l^{(m)} - f_k^{(m)} f_l^{(0)}\right] K_{in}^{kl} d\Omega =$$
$$= -\frac{1}{2} \sum_{k,l,n} \iiint_{-\infty}^{+\infty} \left[f_l^{(1)} f_n^{(m-1)} + \ldots + f_l^{(m-1)} f_n^{(1)} - f_k^{(1)} f_l^{(m-1)} - \ldots - f_k^{(m-1)} f_l^{(1)}\right] \times$$
$$\times K_{in}^{kl} d\Omega - \frac{\partial_{m-1}}{\partial t} f_i^{(0)} - \ldots - \frac{\partial_0}{\partial t} f_i^{(m-1)} + \mathbf{u} \frac{\partial}{\partial \mathbf{r}} f_i^{(m-1)}, \quad i = 1, \ldots, s. \tag{1.22}$$

It can be shown that (1.22) is a system of integral equations, a Fredholm equation with symmetrical kernels.

2. DISTRIBUTION FUNCTION AND CURRENT FUNCTION IN THE FIRST APPROXIMATION.

For the first approximation, we have the equations:

$$\frac{1}{2}\sum_{k,l,n}\iiint_{-\infty}^{+\infty} f_l^{(0)} f_n^{(0)}\left[\Phi_l^{(1)}+\Phi_n^{(1)}-\Phi_k^{(1)}-\Phi_i^{(1)}\right]K_{in}^{kl}\,d\Omega = \\ =\frac{\partial_0 f_i^{(0)}}{\partial t}+\mathbf{u}\cdot\frac{\partial}{\partial \mathbf{r}} f_i^{(0)},\quad i=1,\ \ldots,\ s, \tag{2.1}$$

where

$$\Phi_i^{(1)}=\frac{f_i^{(1)}}{f_i^{(0)}}.$$

In the right part of equation (2.1) let us change to eigen velocities. Utilizing the fact that $f_i^{(o)} = f_i^{(o)}(y_o, \ \ldots \ , y_{r+4}, U)$, we can write:

$$\frac{D_0 \ln f_i^{(0)}}{Dt}=\sum_{l=0}^{r+4}\frac{\partial \ln f_i^{(0)}}{\partial y_l}\cdot\frac{D_0 y_l}{Dt},$$

and in turn

$$\frac{D_0 y_l}{Dt}=\sum_{k=0}^{r+4}\frac{\partial y_l}{\partial x_k}\,\frac{D_0 x_k}{Dt}.$$

Determining $\frac{\partial y_1}{\partial x_k}$ from the system (1.15) and substituting $\frac{D_o x_k}{Dt}$ from (1.18), we obtain the following equations for the distribution function in the first approximation:

$$\frac{1}{2}\sum_{k,l,n}\iiint_{-\infty}^{+\infty} f_l^{(0)} f_n^{(0)}\left[\Phi_l^{(1)}+\Phi_n^{(1)}-\Phi_k^{(1)}-\Phi_i^{(1)}\right]K_{in}^{kl}\,d\Omega = \\ =-f_i^{(0)}\Big\{\mathrm{U}\cdot\left(\frac{m_i}{\rho}-\frac{\Delta_{0,\,r+1}}{\rho\Delta}\,\frac{x_0 m_i}{y_{r+1}}+\frac{\Delta_i^{(0)}}{\Delta}\right)\frac{\partial x_0}{\partial \mathbf{r}}+ \\ +\mathrm{U}\cdot\sum_{k=1}^{r+4}\left(\frac{\Delta_i^{(k)}}{\Delta}-\frac{x_0 m_i}{\rho y_{r+1}}\,\frac{\Delta_{k,\,r+1}}{\Delta}\right)\frac{\partial x_k}{\partial \mathbf{r}}-\frac{m_i}{kT}\,\overset{*}{\overline{\mathrm{UU}}}:\overline{\frac{\partial}{\partial \mathbf{r}}\,\mathbf{u}_0}\Big\}. \tag{2.2}$$

From (2.2 we obtain the following form of the distribution function:

$$f_i^{(1)} = \alpha \exp\left\{ \sum_{\lambda=1}^{r} K_{\lambda i}\mu_\lambda - \frac{1}{kT}\left(\frac{1}{2} m_i U^2 + \varepsilon_i\right) + \nu J_i \right\} \left[1 - \sum_{k=0}^{r} A_{ik}(r, U, t) \cdot \frac{\partial N_k}{\partial r} - \right.$$
$$- A_{i,r+1} \cdot \frac{\partial E}{\partial r} - A_{i,r+1} \cdot \frac{\partial I_{0x}}{\partial r} - A_{i,r+3} \cdot \frac{\partial I_{0y}}{\partial r} - A_{i,r+4} \cdot \frac{\partial I_{0z}}{\partial r} -$$
$$- \overline{B}_i(r, U, t) : \frac{\partial u_0}{\partial r} + \alpha_1^{(1)} + \sum_{\lambda=1}^{r} K_{\lambda i}\mu_\lambda^{(1)} + \vec{\alpha}_2^{(1)} m_i \cdot U +$$
$$\left. + \vec{\nu}^{(1)} \cdot J_i + \alpha_3^{(1)}\left(\frac{1}{2} m_i U^2 + \varepsilon_i\right) \right], \quad i = 1, \ldots, s. \tag{2.3}$$

Here the coefficients $A_{ik}(r, U, t)(k=0, \ldots, r+4)$ and $\overline{B_i}(r, U, t)$ represent particular solutions of the following systems of integral equations:

$$\frac{1}{2} \sum_{k,l,n} \iiint_{-\infty}^{+\infty} f_i^{(0)} f_n^{(0)} \left(A_{nj} + A_{ij} - A'_{lj} - A'_{kj} \right) K_{in}^{kl} d\Omega =$$
$$= f_i^{(0)} \left(\frac{\Delta_i^{(j)}}{\Delta} - \frac{N_0 m_i}{pT} \frac{\Delta_{j,r+1}}{\Delta} + \delta_{j0} \frac{m_i}{p} \right) U, \tag{2.4}$$
$$j = 0, \ldots, r+4; \quad i = 1, \ldots, s$$

$$\frac{1}{2} \sum_{k,l,n} \iiint_{-\infty}^{+\infty} f_i^{(0)} f_n^{(0)} \left(\overline{B}_i + \overline{B}_n - \overline{B}'_l - \overline{B}'_k \right) K_{in}^{kl} d\Omega = f_i^{(0)} \frac{m_i}{kT} \overset{*}{\overline{UU}}. \tag{2.5}$$

The quantity

$$\Delta = \frac{D(F_0, \cdots, F_{r+4})}{D(y_0, \cdots, y_{r+4})}$$

is designated by Δ in expressions (2.2)-(2.4). The determinant $\Delta_i^{(j)}$ is obtained from the determinant Δ, if the $j^{\underline{th}}$ line is replaced by:

$$\frac{1}{\alpha}, \; k_{1i}, \ldots, k_{ri}, \; \frac{1}{kT^2}\left(\frac{m_i U^2}{2} + \varepsilon_i\right), \; J_{ix}, J_{iy}, J_{iz},$$

and $\Delta_{j,r+1}$ is the algebraic complement of the $j^{\underline{th}}$ line and of the $(r+1)^{\underline{th}}$ column of the determinant Δ.

$$\frac{m_l}{kT}\overset{*}{\mathrm{UU}}=\begin{pmatrix} \frac{m_l}{kT}U_x^2-1+p\frac{\Delta_l^{(r+1)}}{\Delta} & \frac{m_l}{kT}U_xU_y & \frac{m_l}{kT}U_xU_z \\ \frac{m_l}{kT}U_yU_x & \frac{m_l}{kT}U_y^2-1+p\frac{\Delta_l^{(r+1)}}{\Delta} & \frac{m_l}{kT}U_yU_z \\ \frac{m_l}{kT}U_zU_x & \frac{m_l}{kT}U_zU_y & \frac{m_l}{kT}U_z^2-1+p\frac{\Delta_l^{(r+1)}}{\Delta} \end{pmatrix}$$

p is the hydrostatic pressure. Systems (2.4)-(2.5) are systems of integral Fredholm equations with symmetrical kernels, and the conditions of solvability are fulfilled for them. Thus A_{ij} and $\overline{B}_i$, in the particular case of an isotropic mixture, are given in the form: *

$$\mathbf{A}_{ij}=A_{ij}(U)\,\mathbf{U},\ \overline{\mathbf{B}}_l=B_l(U)\overset{*}{\mathbf{UU}}.$$

The coefficients $\alpha^{(1)}$, $\mu_\lambda^{(1)}$ ($\lambda = 1, \ldots, r$) $\bar{\alpha}_2^{(1)}$, and $\alpha_3^{(1)}$ are determined from equations (2.6) and (2.7):

$$\sum_i\int_{-\infty}^{+\infty}f_i^{(0)}\left[\alpha_1^{(1)}+\sum_{\lambda=1}^{r}K_{\lambda i}\mu_\lambda^{(1)}+\alpha_3^{(1)}\left(\frac{1}{2}m_iU^2+\varepsilon_i\right)-\right.$$
$$\left.-B_l(U)\sum_{\alpha=1}^{3}\left(\frac{m_l}{kT}U_\alpha^2-1+p\frac{\Delta_l^{(r+1)}}{\Delta}\right)\frac{\partial u_{0\alpha}}{\partial x_\alpha}\right]\Psi_i^\lambda d\mathbf{U}=0,\qquad (2.6)$$
$$\lambda=1,\ldots,r+1.$$

Where Ψ_i^λ is understood to designate the quantities:

$$\Psi_i^\lambda=\begin{cases}1, & \lambda=0,\\ K_{\lambda i}, & \lambda=1,\ldots,r,\\ \frac{1}{2}m_iU^2+\varepsilon_i, & \lambda=r+1,\end{cases}$$

$$\sum_i\int_{-\infty}^{+\infty}f_i^{(0)}\left[\vec{\alpha}_2^{(1)}m_i-\sum_{k=0}^{r+1}A_{ik}\frac{\partial x_k}{\partial \mathbf{r}}\right]m_i\mathbf{U}^2d\mathbf{U}=0.\qquad (2.7)$$

The heat flux, mass flux, and the stress tensor are given by the following expressions to the first approximation:

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* If $I_0 \neq 0$, then A_{ij} and $\overline{B}_i$ have a more complex form. In this connection, supplementary terms appear in the expressions for the fluxes.

$$q = -\left\{ \sum_{k=0}^{r} \lambda_k \frac{\partial N_k}{\partial r} + \lambda_{r+1} \frac{\partial E}{\partial r} \right\}. \tag{2.8}$$

$$\widetilde{U}_i = -\frac{1}{n_i} \left\{ \sum_{k=0}^{r} L_{ik} \frac{\partial N_k}{\partial r} + L_{i,r+1} \frac{\partial F}{\partial r} \right\}. \tag{2.9}$$

$$\overline{P} = p\overline{V} - 2\mu \frac{\overset{\circ}{\partial u_0}}{\partial r} + \beta \left(\frac{\partial}{\partial r} \cdot u_0 \right) \overline{V}, \tag{2.10}$$

where

$$\lambda_k = \sum_i \int_{-\infty}^{+\infty} f_i^{(0)} \widetilde{A}_{ik}(U)\, U_x^2 \left(\frac{1}{2} m_i U^2 + \varepsilon_i \right) dU,$$

$$k = 0, \ldots, r+1,$$

$$L_{ik} = \int_{-\infty}^{+\infty} f_i^{(0)} \widetilde{A}_{ik}(U)\, U_x^2\, dU.$$

$$k = 0, \ldots, r+1;\ i = 1, \ldots, s,$$

$$\mu = \frac{1}{10} \sum_i \int_{-\infty}^{+\infty} m_i f_i B_i(U) \left(\overset{\circ}{UU} : \overset{\circ}{UU} \right) dU,$$

$$\beta = \frac{1}{3} \sum_i m_i \int_{-\infty}^{+\infty} f_i^{(0)} U_x^2 \left[B_i(U) \left(\overset{*}{UU} : \overline{V} \right) + \sum_\lambda D_\lambda K_{\lambda i} + D_{r+1} \left(\frac{1}{2} m_i U^2 + \varepsilon_i \right) \right] dU.$$

Here μ is the coefficient of shear viscosity, and β is the coefficient of dilatational viscosity.

$\widetilde{A}_{ik} = A_{ik} + C_k$ $\left(C_k (k = 0, \ldots, r+1) \right.$ are the coefficients for $\frac{\partial x_k}{\partial r}$ in the representation of $\alpha_2^{(i)}$ by $\left. \frac{\partial x_k}{\partial r} \right)$; the D_k are proportionality coefficients between $\sum_{\alpha=1}^{3} \frac{\partial u_{0\alpha}}{\partial x_\alpha}$ and $\alpha_1^{(i)}$, $\mu_\lambda^{(i)}$ ($\lambda = 1, \ldots, r$), $\alpha_3^{(i)}$ respectively.

In connection with the presence of internal degrees of freedom in formulas (1.13), the transport tensor of the internal moment of momentum is:

$$\overline{M} = \sum_i n_i \overline{\widetilde{U}_i J_i}. \tag{2.11}$$

In addition, in the presence of chemical reactions, the vector currents of atoms of a given type R_λ can be easily examined:

$$R_\lambda = \sum_i n_i \widetilde{U}_i K_{\lambda i}. \tag{2.12}$$

Substituting the expressions for the fluxes, (2.8), (2.9), (2.10), (2.11), and (2.12), in the transport equations, we can write the equations of hydrodynamics which correspond to the first approximation of the distribution function. If several new notations are introduced, the hydrodynamic equation system will take the following form:

$$\left.\begin{array}{c}\rho\frac{D\mathbf{u}_0}{Dt}+\frac{\partial}{\partial x}\vec{\tau}_x+\frac{\partial}{\partial y}\vec{\tau}_y+\frac{\partial}{\partial z}\vec{\tau}_z=0,\\ \frac{DE}{Dt}+E\operatorname{div}\mathbf{u}_0+\frac{\partial}{\partial x}t_x+\frac{\partial}{\partial y}t_y+\frac{\partial}{\partial z}t_z+\vec{\tau}_x\frac{\partial\mathbf{u}_0}{\partial x}+\vec{\tau}_y\frac{\partial\mathbf{u}_0}{\partial y}+\vec{\tau}_z\frac{\partial\mathbf{u}_0}{\partial z}=0,\\ \frac{DN_m}{Dt}+N_m\operatorname{div}\mathbf{u}_0+\frac{\partial S_{mx}}{\partial x}+\frac{\partial S_{my}}{\partial y}+\frac{\partial S_{mz}}{\partial z}=0,\\ m=0,\ldots,r,\end{array}\right\} \tag{2.13}$$

where

$$\left.\begin{array}{l}\tau_{\alpha\alpha}=p-2\mu\frac{\partial u_{0\alpha}}{\partial x_\alpha}+\left(\frac{2}{3}\mu+\beta\right)\operatorname{div}\mathbf{u}_0,\\ \tau_{\alpha\gamma}=\tau_{\gamma\alpha}=-\mu\left(\frac{\partial u_{0\alpha}}{\partial x_\gamma}-\frac{\partial u_{0\gamma}}{\partial x_\alpha}\right),\\ \qquad\alpha=1,2,3,\ \gamma=1,2,3;\end{array}\right\} \tag{2.14}$$

$$\mathbf{t}=\lambda_{r+1}\frac{\partial E}{\partial \mathbf{r}}+\sum_{k=0}^{r}\lambda_k\frac{\partial N_k}{\partial \mathbf{r}}. \tag{2.15}$$

$$\mathbf{S}_m=-\left(\sum_i K_{mi}L_{i,r+1}\right)\frac{\partial E}{\partial \mathbf{r}}-\sum_{k=0}^{r}\left(\sum_i K_{mi}L_{ik}\right)\frac{\partial N_k}{\partial \mathbf{r}}. \tag{2.16}$$

The system of equations (2.13) is closed, since

$$\rho=\sum_i m_i\int_{-\infty}^{+\infty}f_i^{(0)}d\mathbf{U}=\sum_\lambda m_\lambda N_\lambda,$$

$$p=\sum_i\int_{-\infty}^{+\infty}f_i^{(0)}m_iU_x^2d\mathbf{U}=kN_0T,$$

because T is a function of N_0, N_1, ... , N_r and E, and all the coefficients μ, β, λ_k and L_{ik} prove to be functions of N_0, N_1, ... , N_r and E in the last analysis.

3. MIXTURE WITHOUT CHEMICAL REACTION

In the absence of chemical reactions, not only a number of atoms of each type survive the collisions, but also chemical types of colliding molecules. In connection with this, new additive collision invariants appear, which make it possible to greatly simplify the method given above for solving the problem.

The conservation law for a chemical type can be written as follows:

$$(\delta_{jq})_i + (\delta_{j_1q})_n = (\delta_{j'q})_k + \left(\delta_{j_1'q}\right)_l. \tag{3.1}$$

Here δ_{ij} is the Christoffel symbol, i, n, k and l are the indexes for the internal state of the molecules of the $j^{\underline{th}}$, $j_1^{\underline{th}}$, $j'^{\underline{th}}$ chemical types respectively, and t is the number of chemical types of the mixture.

It can be seen from equation (3.1) that the identity element and $K_{\lambda i}$ ($\lambda = 1, \ldots, r$) are linearly-dependent on the invariants δ_{iq}.

The case of an isotropic mixture ($J_{ij} = 0$, $I_0 = 0$) without chemical reactions was studied in the works (Ref. 2 and Ref. 3); it is true that this is somewhat different from the above division of the operator $D_i^{(1)} f_i$ into linearly-independent parts. It can be shown that the appearance of the Eucken correction in the thermal conductivity coefficient and the dilatational viscosity in the stress tensor is connected with the internal degrees of freedom and chemical reactions.

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